Table 1 Frequency response data and rms responses for first-order wind gust model, full-state feedback, Kalman estimators, and robust Kalman estimators

Control	Sensor location	b	rms control activity δ, deg	$\dot{\delta}$, deg/s	Db	Gain margin rad/s	Db	rad/s	deg	Phase margin rad/s	deg	rad/s	Bandwidth rad/s
Full state	N.A.	N.A.	1.9485	178.14	−8.1	62.2	∞	∞	−63.7	44.3	+82.3	95.8	146
Kalman	28	0.0	2.8158	209.83	−1.8	384.5	+3.7	116.5	−41.5	53.7	+22.8	85.6	406
Robust	28	0.000001	4.0835	177.79	−5.6	61.9	+26.3	13.9	−37.9	49.1	+55.6	80.1	105
Robust	28	0.00001	5.4643	203.27	−6.8	61.6	+28.9	9.8	−46.2	45.9	+71.1	85.9	116
Robust	28	0.0001	7.5578	221.12	−7.5	61.9	+50.7	2.3	−55.2	44.6	+76.6	90.6	133

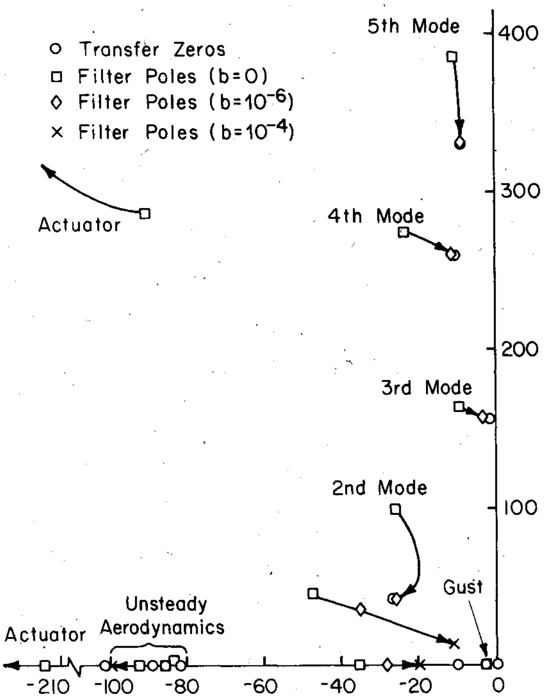

Fig. 2 Locus of filter poles as a function of b.

aeroelastic modes are reasonably well damped and cause no difficulties. The third and fourth modes are lightly damped; however, there is also a lightly damped transfer zero at about the same frequency; consequently, a pole-zero cancellation is effectively obtained for these two modes. There is a considerable difference in the frequency associated with the fifth aeroelastic mode and that of the nearest transfer zero; consequently, no cancellation occurs for this set of poles and zeros. Thus there is a lightly damped filter pole at 385 rad/s, which results in the poor gain margin exhibited at this frequency.

As b is increased, the filter poles move to the open-loop transfer zeros or to infinity as predicted by the theory. Since the order of the compensator transfer function is 18×19, only the pole at -214 s^{-1} moves to infinity. (The filter poles at $-90 \pm j300$ s^{-1} move to transfer zeros of $-304 \pm j1976$ s^{-1} which are located off the figure.) Examination of Fig. 2 also reveals that although Table 1 indicates satisfactory performance, in practice the position chosen for the location of the accelerometer is not acceptable. This is because of the transfer zero located almost on the imaginary axis. As b is increased the third mode filter pole moves to this zero; therefore any error in realization of the robust filter could result in an instability. Thus it is important to know the open-loop transfer zeros when the Doyle-Stein procedure is used.

Conclusions

The Doyle-Stein design procedure has been shown to substantially improve the stability properties of an active flutter controller designed using the linear quadratic Gaussian (LQG) control theory. It is anticipated that the procedure would provide equivalent results in other aeroelastic control problems. No software development is required since the procedure can be performed using existing software for LQG design. However, since the procedure drives the filter poles to the open-loop transfer zeros, it is important that sensor locations be carefully selected. In general it is best to select sensor locations which yield zeros which are distant from the imaginary axis.

Acknowledgment

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An Algorithm for Identification and Analysis of Large Space Structures

E.D. Denman* and J. Leyva-Ramos†
University of Houston, Houston, Texas

I. Introduction

THERE are several identification algorithms currently in use for identifying systems. All of the algorithms can be classified into two principal categories, the least squares

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*Professor, Department of Electrical Engineering.
†Graduate Student, Department of Electrical Engineering.

procedure and the maximum likelihood estimator. Hsia[1] shows that the latter algorithm is equivalent to the least squares estimator. The basic difference between the two is conceptual, not mathematical. There may be a significant difference between the two in computational effort. The estimation problem to be described here is that of computing the mass matrix M, the damping matrix C, and the stiffness matrix K for the structure system defined by

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=Gu(t) \tag{1}$$

and

$$y(t)=Hx(t) \tag{2}$$

where G is the excitation matrix and H is the measurement matrix. It will be assumed that M, C, and K are $n\times n$, that G is $n\times m$, and that H is $l\times n$, with $x(t)$ $n\times 1$ and $y(t)$ $l\times 1$. It is further assumed that $x(t)$ is a vector of node displacements of the structure and $y(t)$ is the measured node displacement. Since the G and H matrices define the output-input relations for transducers, these matrices will be frequency dependent although it will be assumed that these matrices are constant. This assumption is satisfactory provided that the transducers are broadbanded.

The triplet (M,C,K) will be identified by obtaining the Laplace transform variables $Y(s)$, $X(s)$, and $U(s)$ by means of the quadrature formulas of Bellman, Kalaba, and Lockett.[2] The approach to the identification problem differs from other algorithms in that the matrix polynomials Ms^2+Cs+K is directly considered rather than computing eigenvalues and eigenvectors.[3-5] It will be shown that a minimum of measurements is necessary and that measurements of the input and output are taken at a small number of prescribed times. The number of measurements of node displacements must be equal to the order of the quadrature times the number of required variables $Y(s)$ required for identification. The measurement times are set by the quadrature order and are unique for each quadrature order.

II. Quadrature Algorithm for Laplace Transforms

Let $F(s)$ and $f(t)$ represent general functions where $f(t)$ is a transformable vector and $F(s)$ the Laplace transform of $f(t)$. $F(s)$ is given by

$$F(s)=\int_0^\infty f(t)\exp(-st)\,\mathrm{d}t \tag{3}$$

where $f(t)$ is an $n\times 1$ vector. Let r be a new scalar variable, with $r=\exp(-t)$. Therefore

$$F(s)=\int_0^1 r^{s-1}f(r)\,\mathrm{d}r\approx\sum_{i=1}^{N}w_ir_i^{s-1}f(t_i) \tag{4}$$

where Eq. (4) is the quadrature formula. The variables w_i, r_i, and t_i are the weights, roots, and discrete times, respectively. The weights and roots are obtained from orthogonal functions such as Legendre polynomial. The discrete times, t_i, are given by

$$t_i=-\ell n_e(r_i) \tag{5}$$

Details of the above algorithm can be found in Bellman, Kalaba, and Lockett.[2] Weights and roots for $N=3,5,7,...,15$ are given in the above reference with higher orders given by Konrod.[6] w_i, r_i, and t_i for $N=7$ are given in Table 1.

Although the discussion in Ref. 2 is for scalar functions, the theory holds when $f(t)$ and $F(s)$ are vectors. If $f(t_i)$ is an $n\times 1$ vector, then $F(s)$ will be $n\times 1$. The values of the vector $F(s)$ can be obtained by letting $s=1,2,...,q$. It then follows that numerical values of $F(s)$ can be computed from the measured values of $f(t_i)$ where $t_i=-\ell n(r_i)$.

Table 1 Quadrature weight, roots, and times for $N=7$

i	w_i	r_i	t_i
1	0.064742	0.974554	0.025775
2	0.139852	0.870765	0.138382
3	0.190915	0.702922	0.352509
4	0.208979	0.500000	0.693147
5	0.190915	0.297077	1.213762
6	0.139852	0.129234	2.046127
7	0.064742	0.025446	3.671195

III. Quadrature Algorithm and Identification

The quadrature formula is useful for identification of M, C, and K as given in Eq. (1). The Laplace transform of Eq. (1) is given by

$$[Ms^2+Cs+K]X(s)=GU(s) \tag{6}$$

where it is assumed that the initial conditions are zero. Assume that $X(s)$, the node displacements, and $U(s)$, the inputs to the system, have been computed from $x(t_i)$ and $u(t_i)$. It then follows that

$$\begin{bmatrix} X^T(1) & X^T(1) & X^T(1) \\ 4X^T(2) & 2X^T(2) & X^T(2) \\ 9X^T(9) & 3X^T(3) & X^T(3) \\ \vdots & & \\ q^2X^T(q) & qX^T(q) & X^T(q) \end{bmatrix}\begin{bmatrix} M^T \\ C^T \\ K^T \end{bmatrix}=\begin{bmatrix} U^T(1)G^T \\ U^T(2)G^T \\ U^T(3)G^T \\ \vdots \\ U^T(q)G^T \end{bmatrix} \tag{7}$$

where $q>3n$. It is not a difficult task to compute M^T, C^T, and K^T by a least squares algorithm such as the singular value decomposition (SVD) algorithm. It is necessary to impose a restriction on the input such that the input vectors $u(t)$ have linear independence; i.e., $u_i(t_i)$ should not be a linear combination of the other excitations.

The above example is unrealistic since $x(t)$ is the node displacement of all nodes in the structure and is not known. The measurement vector $y(t)$ can be determined, which means that the identification of M, C, and K must be made from $Y(s)$ and $U(s)$, or

$$Y(s)=H[Ms^2+Cs+K]^{-1}GU(s) \tag{8}$$

Since $Y(s)=HX(s)$, it follows that

$$Y(s)=\sum_{i=1}^{N}w_ir_i^{s-1}Hx(t_i) \tag{9}$$

The matrix H is $l\times n$ and $Y(s)$ will be $l\times 1$ when $x(t_i)$ is $n\times 1$. With this restriction, a least squares set of equations can be written with

$$\begin{bmatrix} Y(1) \\ Y(2) \\ \cdot \\ \cdot \\ Y(q) \end{bmatrix}=\begin{bmatrix} w_1r_1H & w_2r_2H & \ldots & w_Nr_NH \\ w_1r_1^2H & w_2r_2^2 & \ldots & w_Nr_N^2H \\ \cdot & \cdot & \ldots & \cdot \\ \cdot & \cdot & \ldots & \cdot \\ w_1r_1^qH & w_2r_2^qH & \ldots & w_Nr_N^qH \end{bmatrix}\begin{bmatrix} x(t_1) \\ x(t_2) \\ \cdot \\ \cdot \\ x(t_N) \end{bmatrix} \tag{10}$$

If H is an identity matrix, then $Y(s)=X(s)$, which indicates that all node displacements are measured. When it is $l\times n$, as in the general case, then an incomplete set of $x(t_i)$ will be identified. The number of node displacements that are needed to identify M, C, and K completely depends on the structure of the triplet (M,C,K). If M is diagonal with identical entries and if C and K are tridiagonal with identical entries on the diagonal with different but identical entries along the super diagonal, then only two measurements are necessary. Suppose that

$$M = \text{diag}\{m_1, m_2, \ldots, m_n\}$$

$$C = \text{tridiag} \left\langle \begin{matrix} & -c_2 & -c_3 & \ldots & -c_n & \\ c_1+c_2 & c_2+c_3 & c_3+c_4 & \ldots & c_{n-1}+c_n & c_n+c_{n+1} \\ & -c_2 & -c_3 & & -c_n & \end{matrix} \right\rangle \tag{12}$$

with a similar structure for K with $m_1 = m_n = m_2 = m_3 = \ldots = m_{n-1}$, $c_1 + c_2 = c_n + c_{n+1}$, $c_2 + c_3 = c_3 + c_4 = \ldots = c_{n-2} + c_{n-1}$, and the same for K. This case will require three measurements, with three elements of $Y(s)$ required provided that $Y_1(s)$ or $Y_n(s)$ is included in the set. This is obvious from $(Ms^2 + Cs + K)X(s) = GU(s)$, since

$$m_1 s^2 x_1(s) + (c_1 + c_2) s x_1(s) - c_2 s x_2(s)$$
$$+ (k_1 + k_2) x_1(s) - k_2 x_2(s) = \bar{u}_1(s) \tag{13}$$

$$m_2 s^2 x_2(s) - c_2 s x_1(s) + (c_2 + c_3) s x_2(s)$$
$$- c_3 s x_3(s) - k_3 s x_1(s) + (k_2 + k_3) s x_2(s)$$
$$- k_3 s x_3(s) = \bar{u}_2(s) \tag{14}$$

where $\bar{u}_1(s)$ and $\bar{u}_2(s)$ are the first two elements from $GU(s)$.

The general case of $m_i \neq m_j$, $c_i \neq c_j$, and $k_i \neq k_j$ for all $i \neq j$ requires that a complete set of measurements be made to identify the triplet.

As an example, let M, C, and K be given by the values in Table 2.

The complete vector $X(s)$ will be required for this example since the elements of M, C, and K differ for all entries. Letting

$$X(s) = [Ms^2 + Cs + K]^{-1} U(s) \qquad G = I \tag{15}$$

the vectors $X(s)$, $s = 1, 2, \ldots, q$ were computed. Equation (7) was then used with SVD to compute the triplet. The given values of M, C, and K were computed with all errors less that one part in 10^4.

A second test using the above example was made to test Eqs. (7) and (10). The values of $X(s)$ were first computed by Eq. (15). These values were substituted into Eq. (10) with $N = 7$ to compute $x(t_i)$. The resulting values of $x(t_i)$ were then used to compute $X(s)$ from Eq. (4). The values of $X(s)$ obtained from Eq. (4) agreed with those obtained directly from $X(s) = [Ms^2 + Cs + K]^{-1} U(s)$ to five digits. All computations were in double precision except for the values of the weights and roots, which were correct to eight digits.

Table 2 Coefficients for sample problem

i	m_i	c_i	k_i
1	1.5	6.	1000
2	0.1	4.5	500
3	0.4	70.2	750
4	2.0	2.4	300
5	0.8	21.6	800
6	1.3	0.9	200

IV. Modification of the Sampling Times

The sampling times of the system for the identification of the system by the quadrature formula are set by the roots r_i. These sampling times may not be desirable for fast responding systems and may lead to numerical difficulties. The sampling times can be modified by using elementary properties of the Laplace transforms. Consider the function $f(at)$ then

$$f(at) = \int_0^\infty e^{-st} f(at)\,dt = \frac{F(s/a)}{a} \tag{16}$$

which has an associated quadrature formula

$$\frac{F(s/a)}{s} = \sum_{i=1}^{N} w_i r_i^{s-1} f(-a \ln r_i) \tag{17}$$

The sampling times are now given by $t_i = -a \ln r_i$. The sampling times are farther apart for $a > 1$ and closer together for $a < 1$.

The weights and roots for the quadrature procedure remain the same. By interlacing the modified sampling times with the original sampling times, additional values of $F(s/a)/a$ can be obtained. This allows more data points to be utilized and should help to reduce errors due to measurement uncertainties and noise.

V. Conclusions

The quadrature algorithm appears to be a useful method for identifying the mass, damping, and stiffness matrices (M, C, and K) arising in the finite-element formulation of structure problems. Studies made to date indicate that the method requires less data than the state-variable approach and that the form of M, C, and K can be utilized to minimize the number of computations.

Acknowledgments

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